High Temperature Behaviour of Positrons in Argon

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The annihilation decay rates and average energy of positrons in argon gas have been calculated using the Boltzmann diffusion equation formalism at high temperatures as a function of electric fields. A comparison is made between the theoretical and experimental results. The agreement obtained is quite reasonable.

1. Introduction

When a swarm of positrons is introduced into a gaseous assembly, there are many processes by which the positrons can arrive at their eventual annihilation fate. Depending on the energy, some positrons form positronium prior to annihilation. However, when the energy is less than the positronium formation threshold, E_{th} (9 eV for argon, 17.8 eV for helium, etc.), then only elastic collisions are important. The study of the behaviour of such positrons in noble gases under the influence of uniform electric, magnetic and temperature fields has been the subject of many recent investigations [1–5]. Most of the studies have been concerned with moderate electric fields because at high fields complications due to positronium formation arise. No such difficulties come into play in high magnetic fields because this field quenches positronium [6]. An electric field enhances the positron life time while a magnetic field has the opposite effect. This is due to the fact that the positron energy is increased in electric and decreased in magnetic fields. The positron energy can also be influenced by the temperature of the gas. Increasing temperature leads to increase in energy, and thus the positron life time is also enhanced. Electric/magnetic fields have no effect on the energy of the gas atoms (being neutral) directly. However, temperature increases the energy of the gas atoms directly much more than the energy of the positrons. Positrons gain energy via collisions with the gas atoms. The question arises how these fields compete to influence the life time of positrons in gases.

The positron behaviour in gases has been analysed via the solution of the Boltzmann diffusion equation [7] although recently the problem has also been studied using the Monte Carlo method [8]. The present paper follows the first approach whereby the positron distribution function, $f(\boldsymbol{v}, t)$, is represented as a sum of two terms as:

$$f(\boldsymbol{v}, t) = f_0(v, t) + \boldsymbol{v} \cdot \mathbf{f}_1(v, t)/v$$

$f_0(v, t)$ is the spherically symmetrical part and $\mathbf{f}_1(v, t)$ indicates asymmetry in the distribution function. $v = |\boldsymbol{v}|$ is the positron velocity. The two term expansion is expected to be valid at least at low electric fields when there are only elastic collisions. We assume that it is also valid at moderately high electric fields. Using this formulation, we have calculated the positron annihilation decay constant (inverse of life time) in argon gas as a function of electric field and temperature employing the positron-atom momentum transfer and annihilation cross-sections calculated by us recently [9]. The temperature range considered is 150–2000 K. The results show good agreement with available experimental annihilation decay constants.

2. Theory

We consider positrons whose energy is less than E_{th}. In this region, the fate of positrons is governed by elastic and annihilation processes only. The distribution function, $f(\boldsymbol{v}, t)$, of positrons moving under the influence of an uniform electric field and temperature in a gas can be obtained by solving the Boltzmann equation. Assuming that the gas assembly is spatially homogeneous, the form of the Boltzmann equation relevant to our system is [10]:

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$$\frac{\partial f(\boldsymbol{v}, t)}{\partial t} + \frac{e}{m} (\boldsymbol{E} \cdot \boldsymbol{\nabla}_v f(\boldsymbol{v}, t)) = \left(\frac{\partial f}{\partial t}\right)_c . \tag{1}$$

Here $\boldsymbol{\nabla}_v$ is the gradient operator in velocity space. $\boldsymbol{E}$ is the electric field. e and m are the positron charge and mass, respectively. $(\delta f/\delta t)_c$ is the collision term. It represents the rate of change of the distribution function in time due to the scattering and annihilation processes. We write the distribution function as:

$$f(\boldsymbol{v}, t) = f_0(v, t) + \frac{1}{v} (\boldsymbol{v} \cdot \boldsymbol{f}_1(v, t)) \tag{2}$$

with the assumption $|\boldsymbol{f}_1| \ll f_0$. By substituting (2) in (1) and simplifying we arrive at [10]

$$\frac{\partial f_0}{\partial t} + \frac{1}{v}\left(\boldsymbol{v} \cdot \frac{\partial \boldsymbol{f}_1}{\partial t}\right) + \frac{e}{m}\left[\frac{\boldsymbol{v} \cdot \boldsymbol{E}}{v} \frac{\partial f_0}{\partial v} \right. \tag{3}$$

$$\left. + \frac{1}{3 v^2} \frac{\partial}{\partial v} (v^2 \boldsymbol{E} \cdot \boldsymbol{f}_1)\right] = \left(\frac{\partial f_0}{\partial t}\right)_c + \frac{1}{v}\left[\boldsymbol{v} \cdot \left(\frac{\partial \boldsymbol{f}_1}{\partial t}\right)_c\right].$$

and equating the coefficients of zeroeth and first order in $\boldsymbol{v}$ one obtains

$$\frac{\partial \boldsymbol{f}_1}{\partial t} + \frac{e\boldsymbol{E}}{m} \frac{\partial f_0}{\partial v} = \left(\frac{\delta \boldsymbol{f}_1}{\delta t}\right)_c = \left(\frac{\delta \boldsymbol{f}_1}{\delta t}\right)_m + \left(\frac{\delta \boldsymbol{f}_1}{\delta t}\right)_a \tag{4}$$

$$\frac{\partial f_0}{\partial t} + \frac{e}{3 m v^2} \frac{\partial}{\partial v} (v^2 \boldsymbol{E} \cdot \boldsymbol{f}_1) = \left(\frac{\delta f_0}{\delta t}\right)_c$$

$$= \left(\frac{\delta f_0}{\delta t}\right)_m + \left(\frac{\delta f_0}{\delta t}\right)_a , \tag{5}$$

where the collision terms have been split into scattering and annihilation terms denoted by the subscripts m and a, respectively. Let $\nu_m(v)$ be the momentum transfer rate and $\nu_a(v)$ the annihilation rate of positrons at velocity v with the gas atoms. In most systems, $\nu_a(v) \ll \nu_m(v)$ [2, 9, 14]. Thus we ignore the second term on the right hand side of (4). Following Holestein [11], the scattering term can be approximated by $-\nu_m f_1$. The solution of (4), at times greater than the slowing down time of positrons in gases, is then

$$\boldsymbol{f}_1 = \frac{-e\boldsymbol{E}}{m \nu_m}\left(\frac{\partial f_0}{\partial v}\right). \tag{6}$$

The procedure to calculate the terms $(\delta f_0/\delta t)_m$ and $(\delta f_0/\delta t)_a$ has been discussed by Chapman and Cowling [10]. Following this, (5) can be transformed into

$$\frac{\partial f_0}{\partial t} = \frac{1}{v^2} \frac{\partial}{\partial v} \tag{7}$$

$$\cdot \left[\frac{-e v^2}{3 m} \boldsymbol{E} \cdot \boldsymbol{f}_1 + \frac{m}{M} \nu_m v^3 f_0 + \frac{k T}{M} v^2 \nu_m \frac{\partial f_0}{\partial v}\right] - \nu_a f_0$$

Here M is the atomic mass and k ist the Boltzmann constant. Use of (6) in (7) leads to

$$\frac{\partial f_0}{\partial t} = \frac{1}{v^2} \frac{\partial}{\partial v} \left[\left\{\frac{a^2 v^2}{3 \nu_m} + \frac{k T}{M} v^2 \nu_m\right\} \frac{\partial f_0}{\partial v}\right.$$

$$\left. + \frac{m}{M} \nu_m v^3 f_0\right] - \nu_a f_0 \tag{8}$$

where $a = e E/m$ is the positron acceleration. Equation (8) is the basic equation for the spherical part of the distribution function of the positrons. The physical significance of the various terms is as follows: changes with time of the function can occur due to the positrons changing their energy due the electric field and temperature as represented by the two terms of the curly brackets; collisions with the gas atoms are represented by the third term in the square brackets and the positron annihilation by the last term. The boundary conditions on f_0 are $f_0(0, t) = \text{constant}$ and $f_0(\infty, t) = 0$.

Positrons reach thermal equilibrium with the gas atoms at times sufficiently longer than the slowing down times. In this region, the time dependence of f_0 can be approximated as $f_0(v, t) \cong f_0(v) e^{-\lambda_a t}$, where λ_a is referred to as the annihilation decay constant or rate and is the reciproal of positron life time. Now (8) becomes after an integration over velocity:

$$\left[\frac{a^2 v^2}{3 \nu_m} + \frac{k T v^2 \nu_m}{M}\right] \frac{\partial f}{\partial v} + \mu \nu_m v^3 f(v)$$

$$= \int_0^v (\nu_a - \lambda_a) v'^2 f(v') \, \mathrm{d}v' \tag{9}$$

where $\mu = m/M$ and the subscript 0 has been dropped for convenience. Simple exact solutions of Eq. (9) which yield f for realistic scattering and annihilation rates are not possible and one has to resort to numerical computing.

We apply this procedure to study the behaviour of slow positrons in argon gas. This gas has been chosen because enough experimental data on this gas are available. Equation (9) has been solved numerically using an iteration-perturbation technique [7]. The data about the scattering and annihilation rates have been taken from [9]. Moreover, we take argon gas to be at normal pressure.

After having obtained the distribution function, the annihilation decay constant can be computed from Eq. (10):

$$\lambda_a = \left[\int_0^\infty \nu_a(v) f(v) v^2 \, dv\right] \left[\int_0^\infty v^2 f(v) \, dv\right]^{-1} \quad (10)$$

and the average energy of positrons from the relation

$$\bar{\varepsilon} = \frac{1}{2}\left[\int_0^\infty v^2 f(v) \, dv\right] \left[\int_0^\infty f v) \, dv\right]^{-1}. \quad (11)$$

$\bar{\varepsilon}$ has been expressed in units of $T_0 = 300\text{K}$.

3. Results and Discussion

We define the reduced decay constant, $\bar{Z}_{\text{eff}} = \lambda_a / \pi r_0^2 c n$, where $r_0 = e^2/m c^2$ is the classical electron radius, c the velocity of light and n the gas density. From now onwards our reference to the decay constant will always mean $\bar{Z}_{\text{eff}}$. The results of the calculations of the decay constant and the average energy are presented in Figures 1 – 4. Let us take a look at each of these separately.

Figure 1 shows the dependence of the decay constant on temperature in the range 150 – 2000 K for the electric fields 0, 10, 20, 30, 40 and 50 Vcm^{-1}. As we move from curve 1 to 6, the field strength increases. At $E = 0$, the change in the annihilation decay constant is around 40% (curve 1) when the temperature is raised from 150 to 2000 K while the same change is about 10% at $E = 50$ Vcm^{-1} (curve 6). Thus, at weaker fields, the effect of temperature is much greater than at higher fields.

Experimental results [12] on the annihilation decay constant in argon gas are available in the temperature range 135 – 574 K at zero electric field. These points are shown in Figure 1. We see that the agreement between the theoretical and experimental data is very good. No experimental results are available in the high temperature range.

Figure 2 presents the dependence of the decay constant on the electric field at three temperatures: 150, 300, 2000 K. Unlike the temperature variations, we find that there is a certain region of the electric field where the decay constant varies rapidly. The width of this region is about 50 Vcm^{-1} for 150 K and 20 Vcm^{-1} for 2000 K. Also, it is apparent that the variations are faster at lower temperatures than at higher temperatures. This is because the electric

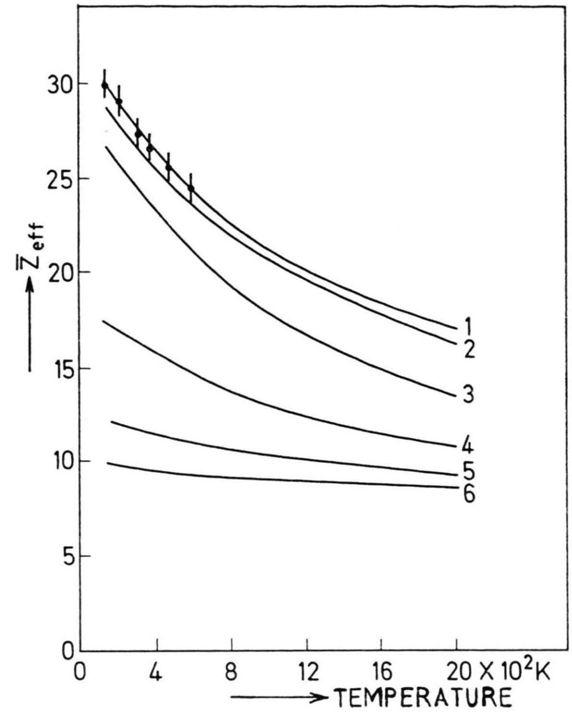

Fig. 1. Variation of annihilation decay constant with temperature at various electric fields. Curves 1–6 are for E = 0, 10, 20, 30, 40, 50 V cm^{-1}, respectively.

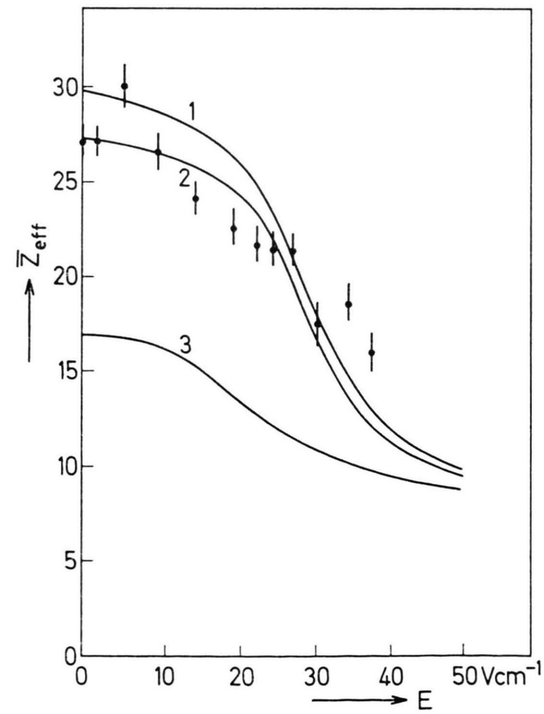

Fig. 2. Dependence of decay constant on electric field. Curves 1, 2, 3 are for T = 150, 300, 2000 K, respectively.

field effect is greater at low temperatures. Dots indicate the experimental data [12]. The values of the decay constant at 300 K and zero electric field are $\overline{Z}_{\text{eff}}$ (experiment) $= 26.6 \pm 0.51$ [12] and 26.77 ± 0.09 [13], while $\overline{Z}_{\text{eff}}$ (theory) $= 27.31$. Thus the agreement is quite good.

Another quantity which can be obtained from the present study is the average energy, $\bar{\varepsilon}$, of the positrons. Once the distribution function has been obtained, the average energy can be calculated from Equation (11). This has been done and the average energy plotted in Fig. 3 as a function of temperature at $E = 0$, 20, 30, 40, 50 V cm^{-1}. In all cases the average energy increases with electric field and temperature. This can be understood from the following argument: We rewrite Eq. (9) after an integration:

$$f(v) = \int_0^v \alpha^{-1} \left[v^{-2} \int_0^v (\nu_a - \lambda_a) v'^2 f(v') \, dv' - \mu \nu_m v f(v) \right] dv \,, \qquad (10)$$

where

$$\alpha = A + B$$

with

$$A = a^2 / 3 \, \nu_m$$

and

$$B = k T \nu_m / M \,,$$

Term A involves the electric field and B the temperature only. A increases with electric field and B with temperature. We see that increasing A and B increase α, and this leads to lowering of the function $f(v)$ which is evident from Equation (10). Looking at Eq. (9), we note that the average energy should increase as $f(v)$ is reduced. That the distribution function is lowered and broadened with increase of the electric field has been shown by our numerical calculations as well [7]. An alternate physical picture indicating that the average energy is increased by the electric field is as follows: for a uniform distribution of positrons, about as many positrons enter a small volume after being scattered in the forward as enter after being scattered in the backward direction. Those scattered backwards are decelerated by the field and thus have a lower energy. Those scattered forwards are accelerated and have higher energy. There are fewer positrons scattered backwards than forwards, and the net effect is an increased average energy.

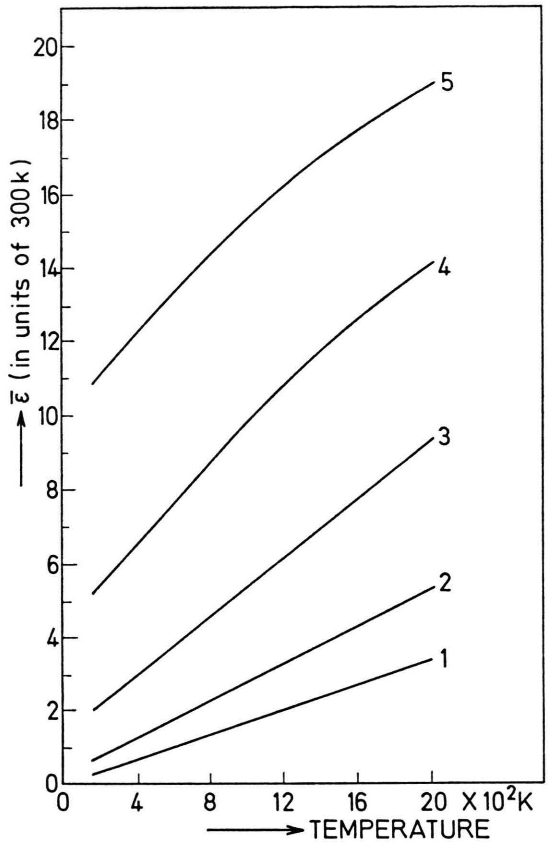

Fig. 3. Effect of temperature on the positron average energy at different fields. Curves 1—5 are for $E = 0$, 20, 30, 40, 50 V cm^{-1}, respectively.

We note from Fig. 3 that (i) the average energy shows a greater dependence on temperature and electric field than the annihilation decay constant; (ii) at lower fields (up to about 30 V cm^{-1}) the variation of the average energy with temperature is linear. This is because term B dominates term A. As the electric field is raised further, the variation of $\bar{\varepsilon}$ with temperature is no longer linear because at higher fields term A becomes equally important. The linear curves of Fig. 3 can be fitted by the straight line $\bar{\varepsilon} = m T + C$ where $(m_1, C_1) = (1.72 \times 10^{-3}, 0)$; $(m_2, C_2) = (2.66 \times 10^{-3}, 0.3)$; $(m_3, C_3) = (4.01 \times 10^{-3}, 1.3)$. The subscripts indicate the curve numbers of Figure 3.

In order to demonstrate the dependence of the average energy on the electric field and to explain the sharp variations of the annihilation decay constant, we have plotted $\bar{\varepsilon}$ as a function of electric field at two temperatures: 300 and 2000 K in Figure 4. We find that there are present rapid variations in the average energy also as a function of electric field. At 300 K (curve 1), $\bar{\varepsilon}$ increases slowly up to $E \approx 15\ \mathrm{Vcm}^{-1}$. Beyond this and up to $E \approx 40\ \mathrm{Vcm}^{-1}$ there is a rapid increase. Now, if we look at curve 2 of Fig. 2, we notice that the annihilation decay constant also suffers a decrease between $E \approx 15$ and $40\ \mathrm{Vcm}^{-1}$. Thus, in the electric field region where the energy undergoes maximum changes, the annihilation decay constant also changes rapidly. Now consider curve 2 of Figure 4. Rapid increase in average energy occurs for $E \approx 10$ to $\approx 30\ \mathrm{Vcm}^{-1}$ and reference to Fig. 2, curve 2, shows that the decay constant varies most in this same region. Thus we see that when the positron lifetime undergoes rapid variations, this corresponds to a rapid gain of their energy.

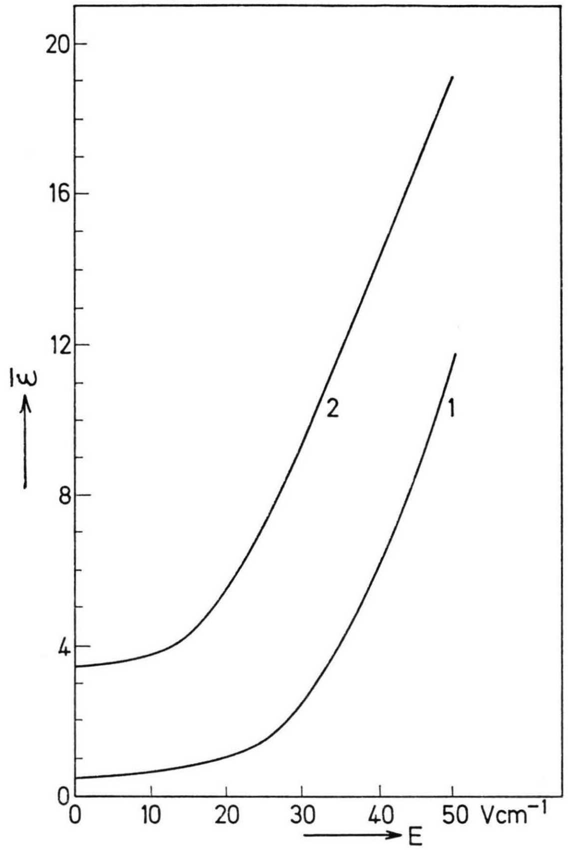

Fig. 4. Variation of average energy with electric field at $T = 300$ K (curve 1) and 2000 K (curve 2). $\bar{\varepsilon}$ in units of 300 k.

4. Conclusions

The conclusions of the present study may be summarized as follows:

(a) It is quite reasonable to represent the positron distribution function by the spherical part. The assumption that asymmetry in the distribution function is small appears to be justified looking at the fact that the calculated results agree quite well with experiment.

(b) The average energy of positrons shows greater variations with applied fields than the annihilation decay constant. It will be interesting and more accurate to measure the average energy experimentally as well.

(c) There exist sharp variations in the annihilation decay constant of argon which can be explained reasonably well by the present data of annihilation and momentum transfer rates.

(d) At high fields and temperatures, the annihilation decay constant is nearly constant while the average energy shows large variations.

[1] P. H. R. Orth and G. Jones, Phys. Rev. **183**, 7 (1969).
[2] R. E. Montogomery and R. W. LaBahn, Can. J. Phys. **48**, 1288 (1970).
[3] K. F. Canter, Contemp. Phys. **13**, 457 (1972).
[4] P. S. Grover and M. P. Srivastava, J. Phys. (B) **5**, 609 (1972).
[5] P. S. Grover and M. P. Srivastava, Can. J. Phys. **53**, 133 (1975).
[6] P. A. Fraser, Adv. Atomic Mol. Physics **4**, 63 (1968).
[7] P. S. Grover, J. Phys. (B) **10**, 2269 (1977).
[8] A. Farazdel and I. R. Epstein, Phys. Rev. **16**, 518 (1977).
[9] P. S. Grover, Appl. Phys. **17**, 109 (1979).
[10] S. Chapman and T. G. Cowling, Mathematical Theory of Non-uniform Gases, Cambridge University Press, London 1970, Chapter 18.
[11] T. Holestein, Phys. Rev. **70**, 367 (1946).
[12] G. F. Lee and G. Jones, Can. J. Phys. **52**, 17 (1974).
[13] P. G. Coleman, T. C. Griffith, G. R. Heyland, and T. L. Killen, J. Phys. (B) **8**, 1734 (1975).
[14] R. P. McEachran, A. G. Ryman, and A. D. Stauffer, J. Phys. (B) **12**, 1031 (1979).